Will's Linear Algebra Project Session 1

UWCCSC Math Circle

November 10, 2025

Problems

Part 1: Linear Independence

For each set, determine if it is linearly independent or linearly dependent. You don't have a write a full proof.

1. **Vector Space:** $\mathbb{R}^3$ Set $S_1 = \{v_1, v_2, v_3\}$

   - $v_1 = (1, 0, 1)$
   - $v_2 = (1, 2, 0)$
   - $v_3 = (0, -2, 1)$

2. **Vector Space:** $\mathcal{P}_2(\mathbb{R})$ **(Polynomials of degree ≤ 2)**   Set $S_2 = \{p_1, p_2, p_3\}$

   - $p_1(x) = x^2 + 1$
   - $p_2(x) = x - 2$
   - $p_3(x) = x^2 + x - 1$

3. **Vector Space:** $\mathbb{C}^2$ **(Complex Vectors)**   Set $S_4 = \{v_1, v_2\}$

   - $v_1 = (1, 1)$
   - $v_2 = (1, i)$

Part 2: Basis and Dimension

1. **Vector Space:** $\mathbb{R}^2$ Does the set $B = \{(1, 2), (3, 1)\}$ form a basis for $\mathbb{R}^2$?

2. **Vector Space:** $\mathbb{R}^3$ From what we just did, we proved that $W = \{(x, y, z) \in \mathbb{R}^3 \mid x+y-z = 0\}$ is a subspace.

   - (a) Find a basis for W.
   - (b) What is the dimension of W?

3. **Vector Space:** $\mathcal{P}_3(\mathbb{R})$

   - (a) What is a standard basis for $\mathcal{P}_3(\mathbb{R})$?
   - (b) What is $\dim(\mathcal{P}_3(\mathbb{R}))$?